Solution to problems of Chapter 2

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2.1 Ask yourself in what direction aerodynamic drag (by definition) acts. Then write the appropriate scaler component of the vector momentum equation in this direction. The body force term for this case is the drag. Solve the equation for this drag.

Solution:
The drag is acting in x-direction, i.e., the horizontal direction.
The integral momentum equation in the x direction is

$$\iint_S (\rho\vec{V}\cdot d\vec{S})u + \iint_{\mathscr{V}} \frac{\partial(\rho u)}{\partial t} d\mathscr{V} = D - \iint_S p(dS)_x \tag{1}$$

Thus,

$$D = -\iint_S (\rho\vec{V}\cdot d\vec{S})u - \iint_{\mathscr{V}} \frac{\partial(\rho u)}{\partial t} d\mathscr{V} - \iint_S p(dS)_x \tag{2}$$

Here D is the total drag force experienced by the body, u is the horizontal velocity component; ρ is the density of the fluid; $(dS)x$ is the infinitesimal area in the x-direction; $\mathscr{V}$ and S are, respectively the volume of the control volume and its bounding surface.

2.2 Similarly, ask yourself in what direction does the lift act. Then write the appropriate scaler component of the vector equation in this direction. The body force term for this case is the lift; solve the equation for this lift.

Solution:
The drag is acting in y-direction, i.e., the vertical direction.
The integral momentum equation in y direction is

$$\iint_S (\rho\vec{V}\cdot d\vec{S})v +_{\mathscr{V}} \frac{\partial(\rho v)}{\partial t} d\mathscr{V} = L - \iint_S p(dS)_y \tag{3}$$

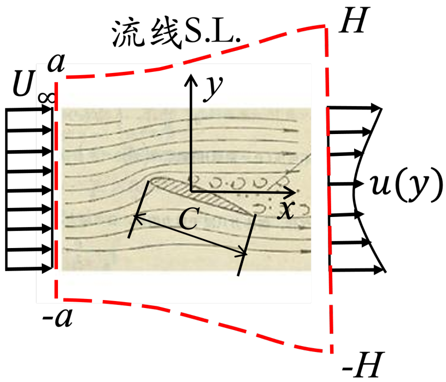
Thus

$$L = \iint_S (\rho\vec{V}\cdot d\vec{S})v + \iiint_{\mathscr{V}} \frac{\partial(\rho v)}{\partial t} d\mathscr{V} + \iint_S p(dS)_y \tag{4}$$

Here L is the total lift force experienced by the body, v is the vertical velocity component; ρ is the density of the fluid; $(dS)_y$ is the infinitesimal area in the y-direction;$\mathscr{V}$ and S are, respectively the volume of the control volume and its bounding surface.

2.3 When the National Advisory Committee for Aeronautics (NACA) measured the lift and drag on airfoil models in the 1930s and 1940s in their specially designed airfoil wind tunnel at the Langley Aeronautical Laboratory, they made wings that spanned the entire test section, with the wing tips butted against the two sidewalls of the tunnel. This was done to ensure that the flow over each airfoil section of the wing was essentially two-dimensional (no wing-tip effects). Such an arrangement prevented measuring the lift and drag with a force balance. Instead, using a Pitot tube, the NACA obtained the drag by measuring the velocity distribution behind the wing in a plane perpendicular to the plane of the wing, i.e., the Pitot tube, located a fixed distance downstream of the wing, traversed the height from the top to the bottom of the test section. Using a control volume approach, derive a formula for the drag per unit span on the model as a function of the integral of the measured velocity distribution. For simplicity, assume incompressible flow.

Solution:



Apply the continuity equation to the control volume shown above. **The control volume is selected such that its top and bottom surface are coincident with streamlines above and below the airflow.**

$$\frac{\partial}{\partial t}\iiint_{\mathscr{V}} \rho d\mathscr{V} + \iint_S \rho\vec{V}\cdot d\vec{S} = 0 \tag{1}$$

For steady incompressible flow $\frac{\partial}{\partial t} = 0$ and $\rho = const$. Equation(1) then becomes

$$\iint_S \rho\vec{V}\cdot d\vec{S} = 0 \tag{2}$$

Thus

$$-\int_{-a}^{a} U_\infty dy \times L + \int_{-H}^{H} u(y)dy \times L = 0 \Rightarrow \int_{-H}^{H} u(y)dy = 2aU_\infty \tag{3}$$

Here U_∞ is the velocity of the free stream. From problem 2.1, the momentum equation in the x-direction is:

$$D = -_S(\rho\vec{V}\cdot d\vec{S})u - \iint_{\mathscr{V}} \frac{\partial(\rho u)}{\partial t} d\mathscr{V} - \iint_S p(dS)_x \tag{4}$$

Also considering steady and incompressible flow, the uniform pressure force on the control surface integrates to zero. Thus the drag force on unit span airfoil is

$$D = -_S(\rho\vec{V}\cdot d\vec{S})u = -\rho\left(\int_{-a}^{a}(-U_\infty^2)dy + \int_{-H}^{H} u^2(y)dy\right) \tag{5}$$

From equation (3) we know $2aU_\infty = \int_{-H}^{H} u(y)dy$, substitute it into equation (5), we have:

$$D = -\rho\left(\int_{-a}^{a}(-U_\infty^2)dy + \int_{-H}^{H} u^2(y)dy\right) = \rho\left[\int_{-H}^{H} u(y)U_\infty dy - \int_{-H}^{H} u^2(y)dy\right] = \rho\int_{-H}^{H}\left[\frac{u(y)}{U_\infty}\left(1 - \frac{u(y)}{U_\infty}\right]dy \tag{6}$$

Here $u(y)$ is the vertical velocity distribution.

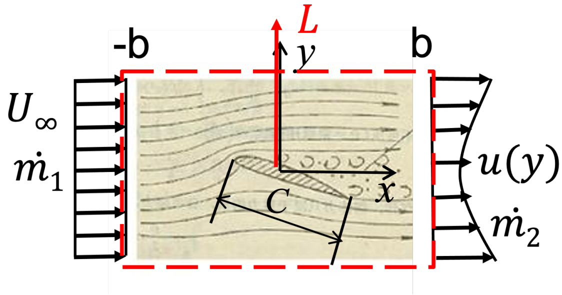
2.4 In the same tests described in problem 2.3, the NACA measured the lift per unit span by measuring the pressure distribution in the flow direction on the top and bottom walls of the wind tunnel. Using a control volume approach, derive a formula for the lift per unit span as a function of the integral of these pressure distributions.

Solution:

Consider steady and incompressible flow. From problem 2.2, the momentum equation in the y-direction can be simplified as:

$$L =_S (\rho\vec{V}\cdot d\vec{S})v + \iint_S p(dS)_y \tag{7}$$

Consider a control volume shown in the below figure



The control volume is selected such that its top and bottom surface coincide with the top and bottom walls of the wind tunnel. Then the lift form per unit span is

$$L = \int_{-b}^{b} p(x)dx + \int_{b}^{-b} p(x)dx \tag{8}$$

Here $p(x)$ is the pressure distribution measured at the top and bottom walls of the wind tunnel.